Judicial fact-finding and the Bayesian method: the case for deeper scepticism about their combination

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Outline

This article argues against the idea of applying the subjective probability theory, associated with Bayes' theorem, in judicial fact-finding. It starts with the already known refutation of the argument that judicial Bayesianism is dictated by logic. Subsequently, it shows that judicial Bayesianism is not even a viable possibility. This is done by identifying the problems intrinsic to judicial Bayesianism, as opposed to rejecting this approach extrinsically on moral or pragmatic grounds, which have already received wide attention. Problems intrinsic to judicial Bayesianism include the following: (1) the problem of criteria; (2) the problem of conversion; (3) the problem of weight; and (4) the conditionalization problem.

The first problem is this: Bayes' theorem can tell judges only how to combine their discrete probability assessments into an integrated decision; judges are thus left free to devise their own criteria for making these assessments. The second problem relates to the standard to be followed by judges in transforming their evaluations of uncertain possibilities into numerical probability estimates. To avoid distortions, subjective probability estimates have to be determined as part of a comprehensive network which encapsulates ab initio both their mutual dependency and their interrelationship as cardinal numbers. Application of Bayes' theorem under these conditions would, however, produce no genuine inferential progress.

The third problem relates to the magnitude of evidence which underlies every
discrete probability assessment and thus determines its relative weight. Bayesianism entails indifference towards this pivotal factor and thus allows judges to combine probability assessments which carry different weights and are therefore incommensurable. The fourth problem lies in processing evidence piecemeal, i.e., item by item, rather than as totality. By not accounting for interrelationships between individual items, this way of updating prior probabilities would systematically reduce the weight of the final assessment. Evidence should therefore be processed as totality. This requirement introduces an all-encompassing conditionalization into each respective relevancy quotient. In a trial involving numerous items of evidence, every such quotient would thus be represented by a unique probabilistic configuration. Unlike simple relevancy quotients, such configurations are normally not determinable by past experience, which makes the Bayesian method less attractive than it appears. More importantly, judges performing such conditionalization must already know the posterior probability, which makes the Bayesian method practically redundant. Furthermore, information sufficient for determining the posterior probability directly would not readily allow judges to decompose this probability into discrete relevancy quotients. In such cases, application of the Bayesian method would be either impossible or suboptimal.

1 Introduction

Judicial reasoning about litigated facts is inherently probabilistic. This reasoning yields probabilities rather than certainties, and avowedly so. At the same time, judges form their decisions about facts without employing Bayes’ theorem or other mathematical methods of probability calculus. This divergence has

1 In this article, 'judges', 'judicial', etc. are meant to include jurors.


3 This observation holds true subject to a few (but notorious) exceptions, such as that dealt with in People v Collins 438 P 2d 33 (1968). See also United States v Shonubi 895 F Supp 460 (1995). In this case, Judge Weinstein (also a prominent American evidence scholar) stated that fact-finding should integrate the inductivist (Baconian) and the Bayesian methods. For sentencing purposes, he extrapolated the amount of heroin, undetectedly imported by the accused on seven occasions, from the amount carried by the accused in his digestive tract on his eighth drug-smuggling trip from Nigeria to the United States and from the general statistics representing the identical practice of other Nigerian drug-smugglers. Judge Weinstein thus used an objectivist probabilistic factor as propensity evidence, which he combined inductively with other evidence presented at the trial. This other evidence, including the accused’s lies, was case-specific and non-statistical. Judge Weinstein’s decision appears to be correct, given that issues pertaining to sentencing were to be determined under the preponderance-of-the-evidence standard. This decision, however, did not integrate the subjectivist Bayesian method with the conventional fact-finding wisdom. Instead, it processed the statistical datum similarly to scientific evidence, by adding it to the list of evidential items inductively supporting the final conclusion as to the quantity of heroin smuggled by the accused. The conventional fact-finding wisdom thus appears to have prevailed.
provoked the ongoing 'probability debate' which focuses, *inter alia*, upon the following normative claim: 'reasoning under uncertainty conducted by judges should follow the Bayesian method of calculating probability'.

This claim has been made in relation to *subjective* evaluations of uncertain possibilities, to which judges are bound to resort. Because contested trials usually involve unique events not amenable to measurement in terms of frequencies or causal propensities, such subjective evaluations are inevitable.

To make the discussion of this claim manageable, I shall focus on its most refined and ambitious version, recently set forth by Bernard Robertson and G.A. Vignaux ('the authors'). According to the authors, application of Bayes' theorem in judicial fact-finding is dictated by logic. This version of judicial Bayesianism is ambitious for an obvious reason. If it is correct, judicial reasoning about facts (as conducted at present) would amount to a persistent logical fallacy.

The aim set for the present article is twofold: first, to demonstrate that the authors' version of judicial Bayesianism is overambitious; and second, to show that judicial Bayesianism is not even a viable possibility. These tasks are undertaken in sections II and III, respectively. Judicial Bayesianism has previously been treated with scepticism originating from pragmatic and moral concerns. Extraneous to Bayesianism itself, each of these concerns can be dispelled by plausible counter-arguments. Section III calls for deeper scepticism towards

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8 These are outlined in Twining & Stein, above, n. 4. See also C.R. Callen, 'Notes on a Grand Illusion: Some Limits on the Use of Bayesian Theory in Evidence Law', *57 Indiana Law Journal* 1, 15 (1982) (arguing that application of the Bayesian method would entail immense computational difficulties in cases involving numerous items of evidence. To this Bayesians would respond that the effort to be made in surmounting such difficulties will be well invested; besides, no other theory of inductive inference can turn complex cases into non-complex ones).
judicial Bayesianism by bringing to the fore its internal problems. These problems show that the Bayesian method would not enhance, and may even undermine, the accuracy of judicial verdicts.

II Is Bayesianism necessitated by logic?

As mentioned at the outset, the authors maintain that this, indeed, is the case. Their argument can be fruitfully understood by the following example. Let it be assumed that X, a person reasonably familiar with English football, is to estimate the outcome of a match which took place between Manchester United and Unknown Amateurs and which he did not witness. X’s knowledge of English football allows him to say that it is highly probable, although not certain, that Manchester United won the match. If so, what probability should be attributed by X to the proposition that Manchester United did not win the match? According to the authors, X’s state of knowledge versus ignorance would logically compel him to ascribe some probability to this proposition. If X were rationally to translate his estimation into betting odds, he would have to assign some value to the possibility that Manchester United had not won the match. To say that X is almost, but not absolutely, certain about his estimation clearly entails an acknowledgement of the chance that this estimation failed to capture the reality. Therefore, if X’s betting odds on United’s victory are, say, 500 to 1, this would logically imply that X estimates the probability of the proposition ‘Manchester United won the match’ as amounting to 500/501. This probability is, indeed, very high, but it falls short of certainty. If X wants to be a coherent bettor, his betting odds on ‘Manchester United did not win the match’ would thus have to be 1 to 500, which would imply the probability of 1/501. In this way, subjective probability estimates are forced to obey the complementational principle, according to which probabilities of any proposition and its negation will always add up to 1. In formal terms:

\[ P(A) + P(\overline{A}) = 1; \quad P(A) = 1 - P(\overline{A}) \]

Let it now be assumed that X was asked, under similar conditions, to estimate the outcome of another match played by Manchester United. He was informed that this match had been played against Uncredited Amateurs and was totally independent of the former one. X decided that the probability of United’s victory would again amount to 500/501. The probability that Manchester United had won

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9 This assumption is made in order to avoid unnecessary complications in the subsequent probability assessment.
twice would thus have to be estimated by X as 250,000/251,001. X is driven by logic to this conclusion, because his chances of not capturing the reality by one of his estimations have been doubled. The probability of the proposition 'Manchester United had not won at least one of the two matches' would consequently have to be estimated by X as 0.004. Subjective estimations of probability are thus forced to obey two further principles: (1) the multiplicative principle (or the conjunction rule), according to which

\[ P(A \cap B) = P(A) \times P(B) \]

or, if propositions A and B are not mutually independent

\[ P(A \cap B) = P(A|B) \times P(B); \]

(2) the disjunction principle for calculating the probability of alternative possibilities,

\[ P(A \cup B) = P(A) + P(B), \]

or, if the events covered by propositions A and B may occur simultaneously,

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]

which would prevent double-counting.

If we supplement all this with the both logical and intuitive principle of transitivity,\(^\text{10}\) the claim put forward by the authors would appear to be irresistible. What remains to be done in order to prove this claim, is simply to derive Bayes' theorem:

\[ P(A \cap B) = P(B|A) = P(A|B) \times P(B) = P(B|A) \times P(A); \]

\[ P(A|B) = \frac{P(A) \times P(B|A)}{P(B)} \]

and then to apply it across the board in all settings which involve uncertainty.

But is this claim really necessitated by sheer logic? Let it now be assumed that X has to estimate the degree of evidential support that he would attribute to each of the following propositions:

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\(^{10}\) According to this well-known principle, to state probability estimates numerically is to reduce them to their common denominator. Therefore, if \(P(A) > P(B); P(B) > P(C)\), then \(P(A) > P(C)\).
(1) Manchester United had won the match against Unknown Amateurs;
(2) Manchester United had won the match against Uncredited Amateurs.

In expressing his estimations, X was also requested to use a 0-100 scale. X’s familiarity with English football gives rise to the following generalization: ‘Teams consisting of highly skilled professionals normally defeat amateur teams’. This generalization has very strong evidential support, but it still falls short of certainty. This is so because its informational base is incomplete, which is true of any generalization that may be used in practical matters. Aware of this limitation (which characterizes any induction”) X decides that evidential support for both proposition (1) and proposition (2) amounts to 98.

Subsequently, X is asked to estimate the degree of evidential support for propositions:

(3) Manchester United had not won the match against Unknown Amateurs;
(4) Manchester United had not won the match against Uncredited Amateurs.

X finds out upon reflection, that his knowledge of English football lends no support to either of these propositions. He is aware of no match that has been won by an amateur team when played against strong professionals. At the same time, he recognizes that his information is incomplete. X then embarks upon the following reasoning:

(a) Evidential support for propositions (1) and (2) is very strong, but not full, as the incompleteness of my information needs to be taken into account. To ascribe full evidential support to those propositions, without accounting for this incompleteness, would amount to procreating knowledge out of ignorance:

(b) Propositions (3) and (4) have no evidential support whatsoever, as I am aware of no game where amateurs have surprised highly skilled professionals. To ascribe positive evidential support to one of those propositions merely because the existing information is incomplete, would thus also amount to procreating knowledge out of ignorance.

To be sure, there is a chance that proposition (1) or (2) is false and, correspondingly, that proposition (3) or (4) is true. However, calculus of chance

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11 Provided, of course, that induction is intended to amplify the existing knowledge, rather than merely restate it in one form or another. For this distinction see L.J. Cohen, An Introduction to the Philosophy of Induction and Probability, (Oxford: Oxford University Press, 1988), pp. 1-4.
and assessment of evidential support are two different frameworks of reasoning. The former framework entails an a priori commitment to the complementational principle, which is not the case with the latter framework. Under the latter framework, non-evidence does not count. Scanty evidential support for proposition A does not entail, eo ipso, the existence of massive support for not-A, or vice versa.

Accordingly, if X were to evaluate the evidential support for the proposition 'Manchester United has won twice', his conclusion would remain unchanged. He would have to estimate this support as not falling below 98. This shows that in assessing evidential support, as distinguished from calculating chances, the multiplicational principle is also inapplicable. By committing himself to both proposition (1) and proposition (2), X takes upon himself an increased chance of not capturing the reality. This, however, does not erode the evidential data supporting each of those propositions and, correspondingly, their conjunction.

Calculation of chances and assessment of evidential support are more than just different. They are, in fact, incommensurable and thus cannot be combined into a single framework of reasoning. The former kind of reasoning, known as 'Pascalian', is founded upon aleatory thinking about uncertainties. Focusing upon chances, this reasoning postulates that the event in question may, and may not, occur (or may, and may not, have occurred). Subsequently, it derives its odds in favour of the resulting binary possibilities by confining itself to the evidence at hand, however incomplete it may be. All this involves an artificial postulation of informational closure, which is not the case with the latter kind of reasoning. Commonly known as 'Baconian', this reasoning focuses upon the amount of evidence favourable to the examined hypothesis. These two kinds of reasoning have been lucidly depicted by Jonathan Cohen:

Baconian probability-functions . . . deserve a place alongside Pascalian ones in any comprehensive theory of non-demonstrative inference, since Pascalian functions grade probabilification on the assumption that all relevant facts are specified in the evidence.

12 Calculation of the chances in the present example would inevitably involve ascription of probative value to propositions like 'Manchester United may not have won one of the matches because Giggs may have decided to upset Schmeichel by scoring own goals'. This would not happen under the evidential support framework.
while Baconian ones grade it by the extent to which all relevant facts
are specified in the evidence.\textsuperscript{13}

Two Swedish scholars have constructed a hypothetical that illustrates both the
difference between the two kinds of reasoning and their incommensurability.
An agent takes upon herself to assess probabilistically the possible outcomes of
three independent tennis matches. The agent's assessments should be made
under the following epistemic conditions:

- \textit{Match A}. The agent is well-informed about the match, which, given the
  players' strengths and weaknesses, promises to be a close one.
- \textit{Match B}. The agent is completely ignorant about the relative capacities of
  the players.
- \textit{Match C}. The agent knows that one of the players is a top professional and
  the other is a rookie, but she doesn't know which is which.

The agent is bound to determine that the probability of each player's winning his
match is equal to 0.5. This estimation, however, fails to capture the evidential
support, which is manifestly different in each of the agent's decisions. Her
decision concerning match A is considerably more evidenced and is thus less
reliant upon ignorance than her decisions with regard to matches B and C.\textsuperscript{14}

detailed accounts and formalisation of Baconian probability see Cohen, above, n. 6, part III; Cohen,
above, n. 11, chs. I, V.


The evidential support factor has been attempted to be integrated within the Pascalian framework
of reasoning. According to David H. Kaye, 'Do We Need a Calculus of Weight to Understand Proof
Beyond a Reasonable Doubt?', \textit{66 Boston University Law Review} 657, 662-72 (1986), evidential gaps can
be accommodated within a probabilistic determination of the disputed facts. Taking \( H \) to represent
the plaintiff's allegations, the plaintiff should arguably be allowed to recover from the defendant if
\( P(H|\neg G) > 0.5 \), when \( E \) stands for the existing evidence and \( G \) denotes the likely impact of the missing
evidence (the evidential gap).

This attempt is unlikely to succeed, as clearly illustrated by the tennis hypothetical. Let \( d \) denote the
difference made by the judges' consideration of the evidential gap: \( P(H|E) - P(H|E\cap G) = d \). This
would entail that \( P(\neg H|E\cap G) - P(\neg H|E) = d \).

Hence: (1) if \( d > 0 \), then \( G \) would increase the probability of the defendant's allegations and
consequently decrease the probability of the plaintiff's case; (2) if \( d < 0 \), then \( G \) would increase
the probability of the plaintiff's allegations and correspondingly decrease the probability of the
defendant's case.

None of these outcomes is warranted because if the evidential gap is real, it should affect both parties
equally. When there are reasons to believe that the missing evidence would have supported one of the
parties, if it were available, those reasons ought to be accommodated within \( E \). The evidential gap
would not be real if, for example, the missing evidence is likely to have been intentionally suppressed
by one of the parties. This additional information would give rise to adverse inferences against the
Arriving at divergent assessments by moving from the Pascalian to the Baconian framework of reasoning, X cannot therefore be accused of violating the canons of logic. Similarly, the existing judicial practices cannot be portrayed as fallacious merely because they do not satisfy the demands of the Bayesian (or other Pascalian) logic. Any such portrayal would be committing an obvious logical fallacy, for it may well be the case — as, indeed, was argued in at least one influential work¹³ — that in resolving issues of fact under uncertainty, judges assess evidential support rather than calculate chances.

Aware of this difficulty, the authors present the Baconian framework of reasoning as intellectually unsustainable:

Baconian probabilities in fact obey their own set of axioms which correspond essentially to the fallacies detected in untutored thinking . . . The rules of[Pascalian] probability are simply dictates of logic. They are rules about how one should think about facts in order to obtain coherent and consistent results . . . [Pascalian] probability . . . relates to thought processes. In order to argue for a different type of probability one must show why facts ought to be thought about in a distinctive way . . . There seems to be no good

14 (continued)

spoliator. In the absence of such information, d would always amount to zero. See D.V. Lindley & R. Eggleston, ‘The Problem of Missing Evidence’(1983) 99 Law Quarterly Review 86. The premise d = 0 postulates that the unknown possibilities, potentially favourable to either the plaintiff or the defendant, are equiprobable. This postulation incomes the ‘principle of indifference’, also known as the ‘principle of insufficient reason’. See J.M. Keynes, A Treatise on Probability (London: MacMillan, 1921), ch. 4. The indifference principle forsakes the introduction of evidential support into probabilistic reasoning, a move that has no apparent justification. For full discussion see A. Stein, ‘The Refoundation of Evidence Law’, 9 Canadian Journal of Law & Jurisprudence (forthcoming in 1996).

James Logue has recently devised a system of subjective second-order probabilities that determine the resiliency of the first-order probability arguments. This system favours subjective differentiation between more and less evidenced probabilities. See J. Logue, Projective Probability (Oxford: Clarendon Press, 1995), pp. 82-95, 150-4. Logue’s innovative insight is of considerable richness and complexity, which make it unambiguous to a parenthetical discussion. For another version of ‘relaxed Bayesianism’ that combines the resiliency factor with first-order probabilities see R. Nozick, The Nature of Rationality (Princeton: Princeton University Press, 1993), pp. 81-93. Any such system would require judges to determine whether there is subjectively sufficient evidence that there is a sufficient subjective probability of the facts in issue. As suggested by Jonathan Cohen, determination of legal liability can safely rest upon the adequate completeness of the evidence that covers the factual ground of the case. The subjectivist mathematical component of the judgment would thus be superfluous. See L.J. Cohen, ‘The Role of Evidential Weight in Criminal Proof’, 66 Boston University Law Review 635 (1986). For an essentially Baconian account that separates probability and weight as distinct objectivist standards of judicial proof see Stein, id.

15 Cohen, above, n. 6.
reason why facts should be thought about differently. There is thus only one kind of probability, namely a measure of strength of belief based upon rational consideration of available information . . . The axioms of probability follow from the fundamentals of rationality, logic and common sense.\textsuperscript{16}

This and related passages contain postulation rather than proof. Strength of belief is, indeed, a pivotal factor, but what justifies an ascription of strength to a belief: the mere chance of capturing the reality, calculated without regard to the broadness of its evidential base, or the extent to which the facts pertaining to the belief are specified in the evidence? The authors provide no reason for treating the former justifiability standard as rationally superior to the latter. No such reason can, indeed, be provided. As has already been mentioned, these two distinct justifiability standards are incommensurable. They escape from any attempt at comparison and can never be fused into a new integrated and thus more comprehensive standard. Within the Baconian framework of reasoning, evidentially unsupported chances are accounted for only \textit{in abstracto}, by recognizing that evidential support can scarcely, if ever, be full. An attempt at calculating such chances in order to treat them as probabilifying would violate the core Baconian principle that prohibits recreation of knowledge out of ignorance. To give up this principle is to break up the whole Baconian framework. Within the Bayesian (or any other Pascalian) framework, varying degrees of evidential support are not allowed to affect the probabilities. To elevate this factor to the rank of a regulating criterion by setting some minimal threshold requirements for evidential support, would give up the core Pascalian principle that treats \textit{any} ignorance as a positive manifestation of probability. To abandon this principle is to break up the whole Pascalian framework.

\section*{III The merits of judicial Bayesianism}

Together with other Bayesians, the authors can step down to a more promising argumentative strategy. Bayesian probability may not be the logic of the law, as suggested by their article's title, but it is certainly a logic that might serve certain legal purposes. A legal system may require that chances attendant upon the lack of knowledge be accounted for by judges. It may require this in order to attain some specified ratio of correct and incorrect adjudicative outcomes in the long

\textsuperscript{16} See above, n. 7, 460-2.
run, a policy that may be general or confined to particular classes of cases. However, even this argumentative strategy is difficult to sustain in relation to subjective probabilities (as distinguished from measurable frequencies and the like). As will be demonstrated below, application of Bayes’ theorem in judicial fact-finding is bound to face severe, and probably insurmountable, obstacles.

Part of the appeal of Bayes’ theorem lies in its universality. As an abstract logical formula, this theorem encapsulates the probabilistic relationship between two or more factual propositions, regardless of their contents. Hence, when H signifies a hypothesis contested at the trial and E stands for a piece of evidence adduced in support of that hypothesis:

\[ P(H|E) = \frac{P(H) \times P(E|H)}{P(E)} \]

The same can be restated as a derivation of the posterior odds from the prior odds and the likelihood ratio.\(^\text{18}\)

Bayes’ theorem thus provides judges with an important insight. It tells them that, when they know the prior probability of H, the probability of E, and the probability of E in cases of H, they can determine the posterior probability of H given E. Sometimes, derivation of the posterior probability from the prior odds and the likelihood ratio may be even more convenient. Subsequently, this posterior probability is used as a prior probability in processing another piece of evidence, and this procedure is repeated time and time again until all pieces of evidence have been taken into account.\(^\text{19}\) Within this framework, \(P(E)\) and \(P(E|H)\) can often be derived from experience. The prior \(P(H)\) thus appears to be the only problematic factor. This well-known problem\(^\text{20}\) is most acute in criminal trials.

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\(^\text{18}\) Namely, as \(O(H|E) = \frac{O(H)}{O(\bar{H})} \times \frac{O(E|H)}{O(E|\bar{H})}\).

\(^\text{19}\) Resort to the Bayesian method would thus entail serious computational difficulties in cases involving numerous items of evidence. See Callen, above, n. 8.

where the presumption of innocence obtains.\textsuperscript{21}

The authors have seemingly downplayed this problem. They regard the presumption of innocence merely as an elliptically stated requirement that guilt be established by rational means and beyond any reasonable doubt.\textsuperscript{22} Consequently, they imply that this requirement would not be violated so long as prior probability is founded upon experience. This idea, however, would strike many as alarming, and rightly so. Ascription of prior probability to a criminal accusation is deeply problematic. Made before considering the evidence, it would involve, in virtually every case, a conjectural derivation of the data from some rough-and-ready generalization. This problem, of course, would not arise if there were firm empirical foundations for prior probabilities of guilt, but normally there are no such foundations.

As the 'beyond reasonable doubt' requirement needs to be satisfied only at the end of the trial, prior probability may arguably be determined after hearing some of the evidence.\textsuperscript{23} This, however, does not solve the problem. In order to avoid double-counting, evidence used by judges in determining the prior probability of guilt would subsequently have to be disregarded. Consequently, this evidence would not be processed through Bayes' theorem, which would undermine the whole approach. Besides, how many items of evidence need to be considered for determining the prior probability? Since it is known that any chosen items will be followed by further evidence, any threshold set for that purpose will be calling for

\textsuperscript{21} This problem has recently been addressed by the Supreme Court of Connecticut in State of Connecticut v Skipper 637 A 2d 1101 (1994). The defendant was convicted of sexually assaulting a child on the basis of paternity statistics obtained by DNA testing. This testing was performed on blood samples taken from the defendant, from the victim allegedly impregnated by him and from the aborted fetus. Linking the defendant with the fetus, it indicated that only one out of 3,497 randomly selected males would have the same genetic traits. This likelihood ratio was converted by the prosecution's expert witness into the posterior probability of 0.9997, an outcome arrived at by postulating that the prior probability of intercourse between the defendant and the victim is 'neutral' and thus amounts to 0.5. This postulation was held to be inconsistent with the presumption of innocence and a new trial was ordered. The court specifically mentioned that: 'Whether a prior probability of 50 per cent is automatically used or whether the jury is instructed to adopt its own prior probability... an assumption is required to be made by the jury before it has heard all of the evidence — that there is a quantifiable probability that the defendant committed the crime. In fact, if the presumption of innocence were factored into Bayes' theorem, the probability of paternity statistics would be useless. If we assume that the presumption of innocence standard would require the prior probability of guilt to be zero, the probability of paternity in a criminal case would always be zero... In other words, Bayes' theorem can only work if the presumption of innocence disappears from consideration'.


\textsuperscript{23} This seems to have been implied by the authors. See above, n. 7, 476.
its own expansion. Prior probability set in this fashion may consequently become posterior.

The prior probability problem can possibly be resolved by unfolding the 'beyond reasonable doubt' requirement. Under this requirement, doubt which is not 'reasonable' would not justify an acquittal. As explained long ago by Lord Denning, law 'would fail to protect the community if it admitted fanciful possibilities to deflect the course of justice'.

This means that sometimes a person may be convicted when innocent. But how many such persons are allowed to be convicted under the 'beyond reasonable doubt' requirement? The social choice besetting this requirement clearly needs to be explicated by the permissible percentage of erroneous convictions. Representing the risk of erroneous conviction to which any person may be exposed, this percentage can be employed as prior probability in all criminal cases. Prior probability would thus be postulated normatively, which appears to be an attractive approach. To postulate prior probability normatively, as, say, 1/100, would certainly be more feasible than to determine it through quasi-empirical conjectures. Bayesian evaluation of evidence may thus take off.

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25 Prior probability as a normative standard can be derived from the accepted disutility ratio:

1. Let $D_g$ and $D_i$ represent, respectively, the disutilities to be incurred by acquitting the guilty and convicting the innocent, and let $P$ denote the probability of guilt;

2. Criminal defendants may thus be convicted whenever: $P > \frac{1}{1 + \frac{D_i}{D_g}}$.

3. i.e., whenever: $P > \frac{1}{1 + \frac{P(D_i)}{D_g}}$.

4. Denoting the risk of erroneous conviction to which any person may justifiably be exposed, prior probability of guilt may thus be determined as constantly amounting to: $1 - \frac{1}{1 + \frac{D_i}{D_g}}$.


Kaplan himself suggested that this principle be used in determining the criminal standard of proof. As for the prior probability of guilt, this, according to him, should be set as being very low, perhaps even as amounting to 1 divided by the number of the country's population. Id., 1085-6. This is an altogether unrealistic approach which would result in too many erroneous acquittals.

In civil trials, the values of $D_g$ and $D_i$ (mutatis mutandis) are usually equal: a mistaken decision for the defendant is as bad as a mistaken decision for the plaintiff. Prior probability for such trials can thus be set as amounting to 0.5. See, however, D.H. Kaye, 'The Probability of an Ultimate Issue: The Strange Cases of Paternity Testing', 75 Iowa Law Review 75, 91-7 (1989) (criticising the idea of postulating the prior probability of 0.5 for civil lawsuits).

In another publication, Bernard Robertson came close to the normative approach by mentioning that: 'A prior probability of zero is simply a formal way of saying "I have an unshakable belief, which cannot be affected by any evidence, that chummy did not do it." Any juror who announced this would presumably be disqualified'.

But will it reach its declared destination? Four interrelated problems, symptomatic of the Bayesian approach to judicial reasoning about facts, induce a rather sceptical answer to this question. These problems would arise in connection with:

1. Judicial criteria for evaluating uncertain possibilities ('the problem of criteria');
2. The standards for converting non-mathematical evaluations of uncertain possibilities into numerical estimates of probability ('the problem of conversion');
3. The standards for determining the adequacy of the informational base upon which judges make their probabilistic evaluations ('the problem of weight');
4. The standards for integrating a series of evidential items in calculating their combined impact upon the ultimate probanda ('the conditionalization problem').

The first two problems exhibit an inherent limitation of the Bayesian method: when applied to subjective probabilities, this method can only be a tool rather than the essence of thought. It can, at best, replicate the already existing subjective judgment, without generating any inferential progress. The last two problems lead to the conclusion that Bayes' theorem cannot fruitfully be employed in judicial trials, even in this limited capacity.

The problem of criteria
Like any other mathematical probability, Bayesian probability is not allowed to break the 0-1 scale. Under uncertainty, any probability would thus amount to more than 0 and less than 1. Hence, given that

\[ P(H | E) = \frac{P(H) \times P(E | H)}{P(E)} \]

subjectively determined probabilities would have to obey another principle, namely:

\[ \frac{P(E | H)}{P(E)} < \frac{1}{P(H)} \]

Let P(H) signify the probability calculated at some intermediate stage of judicial reasoning on the basis of some of the evidence. If P(H) equals, say, 0.12, judges processing a further piece of evidence (E) would not be free to decide that P(E) =
0.1 and \( P(E|H) = 0.9 \). They would not be allowed to make this decision because its outcome violates the rules of probability calculus. The judges would therefore be driven towards revising their assessments.

At first glance, this might appear as a merit rather than a weakness of the Bayesian approach. A system of reasoning that drives judges away from the illogical deserves nothing but commendation. This impression, however, would quickly evaporate once the following questions are asked and attempted to be answered: What exactly had gone wrong? Is it the latest assessments of the probability that are ill-conceived? If not, had the judges fallen into error in one of their previous assessments?

These questions call for a full scale inquiry. If either \( P(E|H) \) or \( P(E) \) are wrongly determined, the trouble might not be great, but error might also lie in \( P(H) \), a figure that integrates the totality of past probabilistic assessments. Each of these assessments might thus be wrong.

In the absence of obvious mathematical miscalculations, an inquiry into these possibilities may be fruitful only in one case. If the judges find something that by their own criteria amounted to an error, they can easily rectify it and reinstall the Bayesian order. Such a trivial case does not merit much attention. In other cases, difficulties to be encountered by the judges in locating the cause of the transpiring absurdity would be enormous. This cause may lie in the criteria endorsed by the judges in evaluating the relevant possibilities as more or less probable than others. These criteria might have been ill-devised, thus producing a distorted probability judgment.

This problem exhibits Bayesian reasoning as a rather non-synergetic coordination of subjective and objective decisional standards. Telling judges how they should update their probability judgments by the incoming evidence, the Bayesian approach pays no regard to the criteria for making those judgments.\(^{26}\) Judges following this approach are left free to devise these criteria subjectively. It is only the merging of discrete probability assessments into an integrated judgment that is meant to be regulated by an externally imposed set of standards. However, as exhibited by the present example, those objectivist standards and the subjectively devised criteria for probability judgments may not live together in peace. In any such case, the former will force the latter into revision, without specifying any guidance for conducting it. Those objectivist standards will ask the judges merely to avoid incoherence. The judges will be told

\[26\] See Cohen, above, n. 11, 67–70.
that if they were to convert their incoherent probability estimates into betting odds, the ensuing bets would be irrational. Those objectivist standards can thus be complied with by judges in a rather remarkable variety of ways. Any subjective estimate of the probability not leading to overtly irrational betting would be good enough. The absurdity transpiring in the present example can therefore be eliminated by any probabilistic adjustment that would install mathematical coherence, however artificial that adjustment may be. A judge who regards an outcome arrived at by the Bayesian method as counterintuitive, would always be able to modify one of her previous estimations in order to arrive at another outcome which she would regard as intuitively correct. The Bayesian framework of reasoning, unquestionably logical in its form, thus turns out to be a logical shell.

This observation was confirmed by an exercise recently conducted by an Australian judge:

As an exercise, I have written a judgment for [a] hypothetical case, which applies Bayes’ theorem . . . It required two assumptions of prior probabilities of hypotheses, and twelve Bayesian steps, each involving two assumptions of numerical probabilities of evidence, given the truth or falsity of hypotheses: twenty six guesses in all. In all twenty six, I found I had virtually no confidence in the numbers I initially selected . . . and I felt I had to check the numbers against the plausibility of the results, and then adjust (and re-adjust) the numbers, in order to arrive at numbers in which I had very slightly more confidence. That is, I had to cheat.\textsuperscript{27}

Logical shells exerting useful constraints upon reasoning processes may still have heuristic value. Unfortunately, no such value can be ascribed to the Bayesian shell when this is used as a framework for judicial fact-finding. This will be demonstrated by the rest of my discussion.

The problem of conversion
Let it be assumed once again that judges have reached an outcome that transgresses one of the canons of mathematical probability. This time, however, they have used adequate criteria for evaluating uncertain possibilities. What could have gone wrong in this case?

There is only one plausible answer to this question. By converting their

evaluations of uncertain possibilities into numbers, the judges have moved from one conceptual scheme to another. Far from being straightforward, this shift entailed commitment not merely to discrete numerical estimates of the probability, but also to their mathematically combined output. In discharging this commitment, it would not be adequate for judges to evaluate \( P(H) \), \( P(E|H) \) and \( P(E) \) independently of each other and subsequently convert their evaluations into mutually unrelated numbers. Judges should evaluate \( P(H) \), \( P(E|H) \) and \( P(E) \), as related to each other, and subsequently convert their evaluations into a scheme of interrelated numerical estimates. In the present case, this procedure must have been ill-performed.

It is difficult to see, however, how this procedure can be performed properly without becoming tautological. Within the framework of empirically ascertainable frequencies, inductions amplifying the existing knowledge can be built up intelligibly (although not always neatly) and thus move the inferential process towards progression. Under this objectivist framework (which, of course, must also determine its ‘confidence interval’ standard\(^2\)), discrete probability estimates are determined by enumeration. Their ultimate product is, of course, affected by their interrelationship as cardinal numbers and, when appropriate, by their mutual dependency, but none of those factors plays a part in the initial enumeration of the data that determines each estimate individually. Within the subjectivist framework of reasoning, probability estimates are not determined by enumeration; they are determined by judicial experience as part of a comprehensive network which should encapsulate their interrelationships and mutual dependency. Probability decisions made without constructing such a network are bound to be distorted. Distortions and absurdities can be prevented only under the comprehensive network conditions. Application of Bayes’ theorem under such conditions would, however, produce no genuine inferential progress.

**The problem of ‘weight’**

Application of Bayes’ theorem as a method of resolving issues of fact in contested trials would be problematic for yet another reason. As already indicated, assessment of evidential support and calculus of chances are mutually incompatible.\(^3\) Under the Bayesian approach, probabilities should be determined through calculus of chances alone. This approach would therefore allow judges to proceed from virtually any informational base, however weak (and thus

\[\text{\footnotesize \text{\cite{29}} See above, n. 14 and the accompanying text.}\]
distorted) it may be. It is, of course, possible to object to this point by arguing that adequacy of the information used in decision-making needs to be secured before plugging this information into the Bayesian system. Bayes' theorem can thus be seen as telling judges how to proceed from an adequate informational base (that is, subject to the predetermined 'confidence interval', 'insensitivity', 'resiliency' or 'robustness' standards\textsuperscript{30}). This objection, however, turns out to be wrong. Different informational bases can never be equally adequate or equally inadequate, and there is no unified standard by which 'adequate' bases can be told from 'inadequate' ones. Adequacy of an informational base is always a matter of 'more or less' rather than 'yes or no'. Depending upon the thickness of its informational base, which admits of different degrees, every probability judgment should thus be assigned its individual degree of strength. Some probability judgments should accordingly be treated as weaker or stronger than others.

This issue can usefully be reformulated in terms provided by Keynes. According to Keynes, probabilistic arguments vary in their weight correspondingly to the magnitude of the evidence upon which they depend:

As the relevant evidence at our disposal increases, the magnitude of the probability of the argument may either decrease or increase, according as the new knowledge strengthens the unfavourable or the favourable evidence; but something seems to have increased in either case — we have a more substantial basis upon which to rest our conclusion. I express this by saying that an accession of new evidence increases the weight of an argument. New evidence will sometimes decrease the probability of an argument, but it will always increase its 'weight'. [T]he weight, to speak metaphorically, measures the sum of the favourable and unfavourable evidence, the probability measures the difference.\textsuperscript{31}

The amount of favourable and unfavourable evidence is a criterion that defies standardisation. Some probability judgments would need more evidence in order


\textsuperscript{31} Keynes, above, n. 14, 77, 84. For further discussion of this essentially Baconian idea see L.J. Cohen, 'Twelve Questions about Keynes's Concept of Weight', (1985) 37 \textit{British Journal for the Philosophy of Science} 263.
to be sound; others would need less. 32 Weight, as remarked by Jonathan Cohen, 'cannot be measured at all, but only compared or, at best, ranked within a fairly narrow 33 field of comparison'. 34

This criterion therefore individuates every probability judgment and consequently denotes the incommensurability of different probability judgments. A conjunction of two or more probability judgments would thus not be governed by the multiplicative principle when this criterion is applied. If \( P(A|B) \) and \( P(B) \) have incommensurable weights, to maintain that \( P(A&B) = P(A|B) \times P(B) \) is to render a probability judgment that has no determinable weight. The outcome of such multiplication would therefore make no sense. Similar considerations would also ban the application of the complementational principle. In science, this problem can often be resolved by conducting controlled experiments which involve standardisation of the data. In judicial trials, where evidence extracted from real-life situations needs to be evaluated 'as is', no such solution is available. Judges should therefore be required to apply some minimal threshold criteria for evidential weight. Probability arguments satisfying these criteria would be treated as commensurable. The Bayesian order would thus be maintained, but not without a price. By not paying full regard to the relative weights of probabilistic arguments, the resulting system would treat weaker and stronger arguments indiscriminately. Why artificially eliminate such an important dimension of uncertain reasoning? 35

The conditionalization problem
This problem will arise in integrating a series of evidential items (found in virtually every trial) within the Bayesian framework. Under this framework,

\[
P(E|H) \]

\[
P(E) \]

is a 'relevancy quotient' (functionally equivalent to the likelihood ratio in a setup of odds 36) which updates the prior \( P(H) \). An additional piece of evidence \( E \) will

32 The utopian 'total evidence' possibility is ignored.
33 I would say, 'extremely narrow'.
34 Cohen, above, n. 11, 109.
35 One should not fail to notice that weight attributable to a probability judgment can never be accommodated within it, e.g., by reducing the probability estimate to which this judgment is to attest. Under the Bayesian framework of reasoning, any reduction in \( P(A) \) would correspondingly increase \( P(\text{not}-A) \). This outcome would clearly be unwarranted, given that both \( P(A) \) and \( P(\text{not}-A) \) rest upon the same informational base. See above, n. 14.
similarly transform $P(H|E_i)$ into $P(H|E_i \cap E_k)$, and so forth. Let $H$ stand for a hypothesis that the defendant is guilty as charged. Let it also be assumed that $E_i$ tends to incriminate the defendant and $E_k$ tends to exonerate him. Should $P(E_i)$ be evaluated by considering the knowledge gained through $E_k$? If it should, then $P(E_i)$ would be lower than it would have been without considering $E_k$. Consequently, the denominator of the relevancy quotient will decrease, which would increase the probability of the defendant's guilt in comparison with the case where $P(E_i)$ is determined without considering $E_k$. If the order of adducing $E_i$ and $E_k$ is reversed, and $P(E_k)$ is chosen to be determined in light of the knowledge acquired through $E_i$, the outcome of the calculation would be different. This time, it would be in favour of the defendant.

The order in which different evidential items are presented should not be allowed to affect the outcome of the trial. Other possibilities of conditionalizing hypotheses upon available evidence need therefore to be considered.

Piecemeal integration

This method prescribes that $P(E_i)$ be determined regardless of the knowledge attainable through $E_k$, and vice versa. Probability of the case as a whole would consequently be this:

$$P(H|E_i \cap E_k \cap \ldots \cap E_j) = \frac{P(H) \times P(E_i|H) \times P(E_k|H) \times \ldots \times P(E_j|H)}{P(E_i) \times P(E_k) \times \ldots \times P(E_j)}$$

Resting on an anomalously fragmented evidential base, this calculation is flawed. To multiply $P(H)$ by a series of isolated relevancy quotients (generated by each evidential item in separation from others) is to determine the posterior probability of the hypothesis without considering the existing information in its entirety. Information ignored by this method would include the impact exerted by $E_i$, $E_k$, $E_j$, and so forth, upon the probabilities forming the relevancy quotients. Accuracy of the verdict would thus be seriously undermined.

This point can be exemplified by the facts of Skipper, a Connecticut case referred to earlier in this article. Those facts will be slightly modified in order to make the example straightforward. A person accused of raping and consequently impregnating a woman, who had later terminated the unwanted pregnancy, has forthrightly denied the allegation. He contends that no intercourse between him and the alleged victim has ever taken place. This defendant was subsequently

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37 See above, n. 21.
subjected to DNA testing, which marked him out as a culprit by the likelihood ratio of 1000/1. If the prior odds in favour of the allegation are set as 1/100, this might not be sufficient for conviction. In such a case, the probability that the allegation is true would amount to 0.91. This probability is very high, but arguably not high enough.

As prescribed by the piecemeal method, the conflicting testimonial accounts given by the complainant and the defendant have now to be considered by the judges in complete isolation from the DNA evidence. Let it be assumed that after having considered these accounts in this fashion, the judges are left in doubt. When isolated from the rest of the evidence, none of these accounts can be supported by reasons capable of making it more probable than the other. These accounts are thus considered by the judges as being equiprobable, so that the probability of the allegation in question remains unchanged. The defendant would consequently have to be acquitted. This unjust and counterintuitive scenario could have been avoided if the complainant’s testimony were not to be isolated from the evidence furnished by the DNA testing.\(^\text{38}\)

All-encompassing conditionalization

This approach holds the following:

**Notation**

\[P_o(H) = \text{the prior probability of } H;\]
\[n = \text{the number of evidential items } (E_1, E_2 \ldots E_n);\]
\[P_n(H) = \text{the posterior probability of } H;\]
\[P(E_{i+1}|H)/P(E_{i+1}) = \text{any selected relevancy quotient.}\]

**The conditionalization formula**

\[
P_n(H) = \frac{P_o(H) \times P_1(E_1|H) \times P_1(E_2|H) \times P_2(E_2|H) \times \ldots \times P_n(E_n|H)}{P_o(E_1) \times P_1(E_2) \times P_2(E_3) \times \ldots \times P_n(E_n)},
\]

\(^{38}\) As I wrote elsewhere in relation to Skipper, ‘If the jury were instructed: “You have to determine the complainant’s credibility in light of the DNA evidence that has shown that only one out of 3,497 males has the same genetic traits as that common to the defendant and the fetus”, would there be any reasonable doubt that the complainant might have picked Skipper at random to falsely accuse him of molesting her? To answer this question, one does not have to be a statistician; no mathematics is required here, just common sense — and here lies the problem for imperialistically inclined Bayesians.’

Allen et al., above, n. 25, 283-284.
when any selected relevancy quotient is determined as follows:

\[
\frac{P(H \cap E_1 \cap E_2 \cap E_3 \cdots \cap E_n) \cap \cdots \cap E_n)}{P(H \cap E_1 \cap E_2 \cap E_3 \cdots \cap E_n) \cap \cdots \cap E_n)}
\]

a formula that takes into account the interrelationships between all items of the evidence.

This, indeed, is the proper way of conditionalizing hypotheses upon evidence. Under this framework, however, every relevancy quotient would be formed by uniquely complex probabilities that cannot usually be determined by general experience. This severe epistemic limitation makes judicial Bayesianism considerably less attractive than it may appear. More importantly, determination of fully conditionalized relevancy quotients would require most substantial knowledge. Judges in possession of such knowledge would usually be able to directly determine the posterior probability of the case.  

39 See R.J. Allen, 'Factual Ambiguity and a Theory of Evidence', 88 Northwestern University Law Review 604, 608 (1994) ('in order to apply statistical concepts to cases, whether frequency or Bayesian, the dependency relationships must be known. This is an example of a general problem ... virtually all theories of evidence require that one know too much').

40 Bas C. van Fraassen demonstrated that conditionalization is not a rationally compelling method for updating prior beliefs by new evidence: it is not necessary for a rational new belief to be logically forced by new experience and prior belief together. See B.C. van Fraassen, Laws and Symmetry (Oxford: Clarendon Press, 1989), pp.160-76. This theory bases fact-finding rationality on a non-Bayesian logic.

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Furthermore, information enabling judges to determine $P(H | E_1 \cap E_2 \cap E_3 \cap \ldots \cap E_i \cap \ldots \cap E_n)$ directly would not always allow them to decompose this probability into discrete relevancy quotients. This, indeed, might happen in many factually disputed cases, where fully conditionalized relevancy quotients are numerous. Should judges abstain from relying upon such information because it is too holistic? If they should, this would equally prevent them from determining heavily conditionalized relevancy quotients. Because judicial fact-finding cannot be halted in indecision, the adherents of judicial Bayesianism are unlikely to favour this prohibition. But they cannot easily reject it either. Holistic conditionalization of posterior probabilities would make the Bayesian method suboptimal.¹⁴

IV Conclusion

As widely acknowledged, statistical analysis of evidence can facilitate the attainment of a number of legal objectives.¹² Principles underlying this analysis should not, however, control the entire process of judicial fact-finding. If elevated to this controlling role, these principles would produce more harm than good. The conventional refusal to apply these principles in settling disputed issues of fact is therefore entirely justified.

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¹⁴ See Allen, above, n. 39 (advocating the same idea by arguing that a holistic approach — featuring comparative en masse evaluation of conflicting factual accounts — is the only one that can properly handle factual ambiguities in contested trials).